

# Vlasov Description Of Dense Quark Matter

A. Bonasera\*

*Laboratorio Nazionale del Sud, Istituto Nazionale di Fisica Nucleare, Via S. Sofia 44, 95123  
Catania, Italy &*

*Cyclotron Institute, Texas A&M University, College Station, TX 77843-3366, USA.*

## Abstract

We discuss properties of quark matter at finite baryon densities and zero temperature in a Vlasov approach. We use a screened interquark Richardson's potential consistent with the indications of Lattice QCD calculations. We analyze the choices of the quark masses and the parameters entering the potential which reproduce the binding energy (B.E.) of infinite nuclear matter. There is a transition from nuclear to quark matter at densities 5 times above normal nuclear matter density. The transition could be revealed from the determination of the position of the shifted meson masses in dense baryonic matter. A scaling form of the meson masses in dense matter is given.

**PACS : 24.85.1p 12.39.Pn**

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\*bonasera@lns.infn.it

## I. INTRODUCTION

The search for a quark-gluon plasma (QGP) is one of the new and most exciting directions in physics at the border between nuclear and particle physics. One way to get to the QGP is via the collisions of relativistic heavy ions and this will be accomplished soon at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven and in the next decade at LHC-CERN [1]. This strategy, the only possible on earth, raises many problems and difficulties which could be resolved with the help of theory. Some of the difficulties are: i) the collision is mainly a non-equilibrium process. Equilibration-even partial- might not be reached during the process and the nuclei could be transparent at very high energies; ii) the properties of the particles change when the density and/or the excitation energy (temperature?) change; iii) it is not clear which 'probes' will give relevant informations on the dynamics of the collision. Thus a theoretical approach must be dynamical and contain the most important features of microscopic approaches like Quantum ChromoDynamics (QCD) which because of its difficulties (numerical and conceptual), has been applied so far to some limited cases such as quark matter at zero baryon density and high temperature (T) [1,2]. Recently [3], we have proposed a dynamical approach based on the Vlasov equation [4,5] to reproduce hadron masses. Some work in the same spirit is discussed in [6]. The approach needs as inputs the interaction potential among quarks which we borrowed from phenomenology i.e. the Richardson's potential [7] and the quark masses which we fitted to reproduce known meson masses such as the  $\pi$ , the  $\phi$ , the  $\eta_c$  etc. When the particles are embedded in a dense medium such as in nuclear matter (NM) the potential becomes screened in a similar fashion as ions and electrons in condensed matter do, i.e. Debye screening (DS) [1,5]. Thus we use a screened potential and search for the relevant parameters which reproduce the properties of NM i.e. a minimum at baryon density  $\rho_0 = 0.16 fm^{-3}$  and an energy per nucleon  $E/nucleon = -16 MeV$ . Then we can study the equation of state (EOS) of quark matter at various baryon densities ( $\rho$ ) and determine the transition from NM to QGP. All the calculations in this paper are at finite  $\rho$  and zero T, but an extension to other cases and to heavy ion collisions is possible and will be discussed in future works. In brief these are our main results:

a) the approach can reproduce the well known properties of NM near the ground state density. This has the important consequence that we can easily extend the calculations to finite nuclei starting from the quark degrees of freedom;

b) the transition between NM and QGP is smooth, we find no evidence for a first order phase transition. Of course the results could change at finite T;

c) some features of QM can be revealed by studying the properties of the hadrons in dense medium. Mesons such as the  $\pi$ ,  $J/\psi$  and  $\gamma(1S)$  can survive in the medium up to a "critical" density which depends on the particles type. We also find that light quark mesons such as the  $\rho$  *never* survive in the medium because of the repulsive chromomagnetic term(c.t.), see eq.(3) below. For those particles that can be bound in QM we can express their masses  $m(\rho)$  in medium as:

$$m_c - m(\rho) \propto (1 - \frac{\rho}{\rho_c})^\beta \quad (1)$$

where  $m_c$  and  $\rho_c$  are the values of the mass and baryon density where the mesons dissolve in medium and  $\beta$  is a fitting parameter. We stress that eq.(1) has nothing to do with the

behavior of an order parameter near the critical point in a second order phase transition. A scaling of the hadron masses in a dense medium has been shown by Brown and Rho [8] based on an effective chiral Lagrangian and the scaling properties of QCD. Also the study of the properties of meson masses in a medium, especially the  $J/\psi$  has been discussed to signal the QGP phase transition at high T [9].

The paper is organized as follows. In section II we review the Vlasov equation and the test particles method. In section III we apply the method to calculate hadron masses while section IV is devoted to the EOS calculations. A brief summary is given in section V.

## II. THE VLASOV EQUATION

We briefly recall some general features of the VE, interested people should look the (not complete) list of references for more details on this equation [1,3–6]. The VE gives the time evolution of the one body distribution function  $f_{qc}(r, p, t)$  in phase-space:

$$\partial_t f_{qc} + \frac{\vec{p}}{E} \cdot \nabla_r f_{qc} - \nabla_r U \cdot \nabla_p f_{qc} = 0 \quad (2)$$

where  $E = \sqrt{p^2 + m_i^2}$  is the energy and  $m_i$  is the quark mass. The underscripts indicate that the distribution function depends on the flavor (q) and color (c) of the quarks.

In agreement to LQCD calculations [2,7] the interacting potential  $U(r)$  for  $q\bar{q}$  is ( $\hbar = 1$ ):

$$U(r) = \frac{8\pi}{33 - 2n_f} \Lambda \left( \Lambda r - \frac{f(\Lambda r)}{\Lambda r} \right) + \frac{8\pi}{9} \bar{\alpha} \frac{\langle \sigma_q \sigma_{\bar{q}} \rangle}{m_q m_{\bar{q}}} \delta(\mathbf{r}) \quad (3)$$

where

$$f(t) = 1 - 4 \int \frac{dq}{q} \frac{e^{-qt}}{[\ln(q^2 - 1)]^2 + \pi^2} \quad (4)$$

( for  $q\bar{q}$  the potential must be decreased of a factor 1/2),  $n_f$  is the number of quark flavors involved and the parameter  $\Lambda$  has been fixed to reproduce the masses of heavy  $c\bar{c}$  and  $b\bar{b}$  systems in [7]. In eq.(3) we have added to the Richardson's potential the c.t., very important to explain the masses of different resonances for light quarks in vacuum. In this work the expectation value of  $\langle \sigma_q \sigma_{\bar{q}} \rangle$  is used depending on the relative spin orientations of the constituent quarks . For instance for the pion this term is equal to -3 while for a  $\rho$  meson it is equal to +1 [2]. The  $\delta$  function is approximated to a gaussian i.e. we make the replacement  $\delta(\mathbf{r}) \rightarrow \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{r^2}{2\sigma^2}}$ . The average strong coupling constant  $\bar{\alpha}$  and the variance  $\sigma$  are adjusted to reproduce the  $\pi$  and  $\rho$  mesons [3]. In dense nuclear matter the c.t. can be neglected.

Numerically the VE equation is solved by writing the one body distribution function as:

$$f_{qc}(r, p, t) = \frac{1}{n_{tp}} \sum_i^N g_r(r - r_i(t)) g_p(p - p_i(t)) \quad (5)$$

where the  $g_r$  and  $g_p$  are sharply peaked distributions (such as delta functions, gaussian or other simple functions), that we shall treat as delta functions.  $N = Qn_{tp}$  is the number

of such terms,  $Q = q + \bar{q}$  is the total number of quarks and antiquarks (for a meson  $Q=2$ ). Actually,  $N$  is much larger than the total quark number  $Q$ , so that we can say that each quark is represented by  $n_{TP}$  terms called test particles(tp). The rigorous mean field limit can be obtained for  $n_{TP} \rightarrow \infty$  where the calculations are of course numerically impossible, even though the numerical results converge rather quickly. Inserting eq.( 5) in the Vlasov equation gives the Hamilton equations of motions for the tp [4]:

$$\begin{aligned}\dot{\mathbf{r}}_i &= \frac{\mathbf{p}_i}{E_i} \\ \dot{\mathbf{p}}_i &= -\nabla_{\mathbf{r}_i} U,\end{aligned}\tag{6}$$

for  $i = 1....N$ . The total number of tp used in this work ranges from 5000 to 50000 with no appreciable change in the results. The equations of motion (6) are solved by using a  $O(\delta t^4)$  Adams-Bashfort method [10].

### III. HADRON MASSES

We recall some results obtained in [3]. In order to calculate hadron masses, we initially distribute randomly the tp in a sphere of radius  $r$  in coordinate space and  $p_f$  in momentum space.  $p_f$  is the Fermi momentum estimated in a simple Fermi gas model by imposing that a cell in phase space of size  $h = 2\pi$  can accommodate at most two identical quarks of different spins. A simple estimate gives the following relation between the quarks density  $n_q$  and the Fermi momentum:

$$n_q = \frac{g_q}{6\pi^2} p_f^3\tag{7}$$

a similar formula holds for  $\bar{q}$ . The degeneracy number  $g_q = n_c \times n_s \times n_f$ , where  $n_c$  is the number of colors and  $n_s$  is the number of spins [1]. For quarks and antiquarks 3 different colors are used red,green and blue (r,g,b) [2]. From the above equation we see that the Fermi momentum for quarks distributed in a sphere of radius 0.5 fm is of the order of 0.5 GeV/c. Thus relativistic effects become important for quark masses less than 1 GeV. For instance for the  $\pi$  case, if we calculate the total energy of the system relativistically we obtain 0.14 GeV (with the parameters choice discussed below), while using the nonrelativistic limit we obtain about 1 GeV! We stress that since the system is properly antisymmetrized at time  $t=0$ fm/c, it will remain so at all times since the VE conserves the volume in phase-space [4].

The masses of the hadrons are given by the total energy of the system. It is the interplay among the Fermi motion, the potential term and the quark masses which determines the masses of the hadrons. For light quarks the chromomagnetic term is important to reproduce the experimental values of the masses. Thus we have adjusted the values of the scale constant and the quark masses to reproduce the data. We found a good fit for mesons by using  $\Lambda = 0.250\text{GeV}$ ,  $\bar{\alpha}_s = 0.225$ ,  $\sigma = 0.5\text{fm}$  [2] and the quark mass values given in Table I. We notice that the parameters and the heavy quark mass values are in good agreement with currently accepted ones [2,12]. The masses of (u,d) quarks in table I is somewhat smaller than the 300 MeV used in many potential models [2]. This is due to the fact that these models are non relativistic but, as we stressed above, because of the Fermi motion, relativistic effects are quite important. In the relativistic approach of [11], where the

Richardson's potential was used as well, the (u,d,s) quark masses have values comparable to ours. The small differences between our results and [11] are most probably due to their relativistic generalization of the Richardson's potential and the different choice of  $\Lambda$ .

The parameters entering our model are essentially fixed on some meson masses. As a consistency check we extended the calculations to the baryons case with the usual modifications of a factor 1/2 to the potential eq.(4), as suggested by LQCD considerations [2].

In figure (1) we display the calculated (open symbols) mass of the resonances vs. the sum of the quark masses (cfr. table I), for mesons (top) and baryons (bottom). The circle symbols refer to attractive, while the squares to repulsive chromomagnetic term in eq.(4) [2]. The corresponding experimental data [12] are given by the full symbols. The overall agreement is quite good in all cases and some predictions for resonances not yet observed are also given.

The results discussed above prove that the VE is suitable to describe the quarks dynamics in hadrons despite the simple form for the two body potential. The approach works rather well for heavy quarks, in that it is able to reproduce the masses and radii of the heavy hadrons (as compared to data or other calculations [2,3,7]) and also the masses of light hadrons. The radii of light quarks systems are underestimated which could be a hint for relativistic corrections to the potential term as discussed for instance in [11]. However some criticism can be raised such as:

i) it is commonly accepted that the main features of light quarks are dictated by chiral dynamics. This is of course a limit of the Vlasov approach where a non-relativistic potential is employed. However, we have shown that a good choice of the parameters can still reproduce the experimental data. One should keep in mind that the purpose of this work is to describe *average* properties that could be observed in nucleus -nucleus collisions and to give some guidance to unveil the occurrence of a phase transition (if any). Thus we need a model that gives similar masses when the particles are isolated, see fig.1, plus the confining properties. Confinement arises here because of the linear term in the Richardson's potential, eq.(3). Furthermore, we require that dense quark matter has similar properties in our approach as nuclear matter near the ground state baryon density, see next section. Since we will be looking for general features such as phase transitions, these features will be similar because the EOS and the rest masses of hadrons are similar;

ii) in the Vlasov approach color neutral states are the result of the initial conditions, i.e. at time  $t=0$  fm/c the system is colorless. The time evolution will keep the system colorless in each region because of the large number of tp. The color degrees of freedom enters only through the Fermi momentum, eq.(7) and they are not contained in the potential term. The only consequence is that if initially one gives 1 or 2 colors for instance to baryons, the Fermi momenta will be higher and so will be the masses;

iii) the initial conditions are given by a Fermi gas. This will result in a total energy (which is constant) for a given initial radius. Of course since the system is finite it will evolve and develop a smooth surface. For long enough times the density distribution will evolve to an equilibrium one, see fig.(1) in [3]. High momentum states of particles arise when two tp are very close and see a strong attraction due to the coulomb term in the potential.

#### IV. DENSE MATTER

The potential becomes screened in dense matter because of the interactions of the many colored quarks which make a colorless object. We incorporate this feature simply by "screening" the Richardson's potential as:

$$U(r) \rightarrow U(r)e^{-\frac{r}{r_d}} \quad (8)$$

where  $r_d$  is the Debye radius given in dense matter at zero temperature by [13]:

$$(r_d)^{-2} = \frac{2\alpha_0}{\pi} \sum_{i=u,d,s..} p_{f_i} \sqrt{p_{f_i}^2 + m_i^2} \quad (9)$$

where  $\alpha_0$  is the strong coupling constant and  $p_{f_i}$  is the Fermi momentum given above, eq.7. The strong coupling constant is in general dependent on the relative distances or momenta of the quarks. Here, we treat  $\alpha_0$  as a constant and fix its value to reproduce the BE of nuclear matter at  $\rho_0$ . We will perform calculations for  $Q = 300$  (u-d) quarks and 30 tp per quark, increasing these numbers does not change the results. The initial conditions are given by randomly distributing the tp in a cube of side L in coordinate space (with periodic boundary conditions) and a sphere of radius  $p_{f_i}$  in momentum space, we fix  $n_f = 3$  [13].

In figure(2) we display the calculated energy per nucleon (1 nucleon=3 quarks) (top part), Debye radius (middle) and energy density of the quarks  $\epsilon$ (bottom) vs.  $\rho/\rho_0$ . The  $\alpha_0$  has been fitted to give the correct BE at  $\rho_0$  for each case. The open symbols refer to the choices  $\Lambda = 100MeV$ ,  $m_u = m_d = 7MeV$  (circles) and  $m_u = m_d = 130MeV$  (squares). For these cases the minimum in the E/nucleon occurs at about twice  $\rho_0$  and deeper BE. The corresponding  $r_D$  are of the order of 1 fm at  $\rho_0$  which is quite a large value. Recall that at such densities the distance among *nucleons* is also of the order of 1 fm and we would expect  $r_D$  to be smaller. Thus we can exclude these sets of parameters from the following considerations. The correct properties of NM at  $\rho_0$  can be obtained for instance for the choice  $\Lambda = 250MeV$ ,  $m_u = m_d = 130MeV$ ,  $\alpha_0 = 1.43$ (triangles-set I) and  $\Lambda = 400MeV$ ,  $m_u = m_d = 7MeV$ ,  $\alpha_0 = 2.26$ (diamonds-set II) [14]. Notice that set I has been employed in section III to reproduce the hadrons masses. The E/nucleon can be estimated by using a Skyrme potential with compressibility  $K = 225 MeV$  (dashed line in figure 2 (top))which gives a good description of NM near the gs density [1,4]. We notice a deviation from the Skyrme result at high densities where the calculated EOS becomes softer because of the transition to the QGP. Another deviation occurs at very low densities which hints to a larger screening in order to reduce the effect of the linear term in the Richardson's potential. However both limits are in the regions where our knowledge of the EOS is rather rough and the Skyrme approximation is questionable. The corresponding  $r_D$  and energy densities (in units of the energy densities obtained in a Fermi gas model  $\epsilon_{FG} = \frac{g_a}{8\pi^2} p_{f_i}^4$  [1]) are displayed in the middle and bottom part of the figure. At  $\rho_0$  the  $r_D$  are about 0.5 fm and the energy density is of course correct. The calculated energy densities cross the Fermi gas result at about 5-7 times normal nuclear matter and tend to the Fermi gas value from below at very high densities because the attractive part of the potential is dominant there. The  $\epsilon$  is a smooth function of  $\rho$  (and  $p_f$ ,eq.7) thus excluding a first order phase transition. Since this plot gives no evidence for a "critical" change of phase, we will loosely speak of nuclear matter

for densities close to the nuclear matter g.s. density and quark matter at high densities. A change from one phase to another can be estimated from the Bag-model [1]. This change will occur when the pressure from the nuclear matter becomes equal to the bag pressure, i.e.  $P_q = B$ . At high densities we can neglect the quark masses, thus the "critical energy density"  $\epsilon_c = 3P_q$  [1]. Using a value of  $B^{1/4} = 0.206\text{GeV}$ , gives  $\epsilon_c = 0.71\text{GeV}$ . This value of energy density is obtained in our calculations at about  $\rho_{cb} = 0.75\text{fm}^{-3}$  for parameters I and II, and it is indicated by the arrow in fig.(2) top.

In order to strengthen this finding we simulate some 'probes' which behave differently depending on the density. If we put a hadron, in particular a meson, in NM, its mass will change because of the DS, eq.(8) [8,9]. In particular we can distribute  $q\bar{q}$  in a sphere of radius  $r_i$  at time  $t = 0\text{fm}/c$  and let it evolve in NM with the screened potential [15]. In figure(3) we show the total energy of the meson (i.e. its mass) vs. the initial  $r_i$  at different densities [1,9], for the  $\pi$  (full circles), the  $\eta_c(1S)$  (triangles) at  $\rho_0$  and the heavier  $\gamma(1S)$  at  $\rho = 1.046\text{fm}^{-3}$  (set II). Notice that the minima occur at smaller radii for heavier quarks which thus probe different densities of the matter. In particular we find different 'critical' densities  $\rho_c$  for the 3 ( $u\bar{d}$ ,  $c\bar{c}$  and  $b\bar{b}$ ) cases where the mesons dissolve. Another important feature is that for light quarks the minima are due to the c.t., infact if we calculate the mass of a  $\rho$  meson in medium (for which the c.t. is repulsive), we find no minima at all densities. In figure (3) we display the results for the  $\rho$  meson (open circles) at  $\rho = 0.0235\text{fm}^{-3}$ . The difference between the mass in medium and the free mass is always positive for the cases where a minimum exists. This difference increases for increasing densities up to the  $\rho_c$  where the minimum disappears. We stress that for the light  $u\bar{d}$  mesons, the calculated radii in vacuum are smaller than data [3], thus in reality the interplay between the Coulombic, the linear and chromomagnetic terms in eq.(3) is different and could change our results, but the qualitative features should remain unchanged.

The results discussed above are summarized in figure (4) in a scaling invariant way. In the figure we show that the differences of the 'critical' mass  $m_c$  minus the mass at density  $\rho$  vs.  $\rho/\rho_c$  fall in the same universal curve which can be parametrized according to eq.(1) and it is given by the full curve in the figure [16]. It is important to notice that our estimate for  $\rho_{cb}$  using the bag model is in good agreement to the critical density obtained when the  $c\bar{c}$  dissolves in the plasma. This particle could give important informations about the density where the QGP is first formed at variance with lighter quark mesons for which the c.t. might be important and for heavier resonances which because of the heavy quark mass are less influenced by the Debye screening (thus dissolve later). It is important to notice however that a direct signature of  $c\bar{c}$  suppression in RHIC might be washed out by the many interactions that the particles might have during the reaction time.

In ending this section we would like to stress the following points:

i) we have calculated the EOS starting from a phenomenological q-q potential which becomes screened in dense matter. Of course if the density goes to zero the screening vanishes and we obtain the confining properties of the potential (due to the linear term). This will be important when dealing with finite nuclei in that it will not be possible to find isolated quarks, but rather quarks must be always grouped in order to have sizable densities. Again the cluster of quarks will be colorless on average because of the large number of t.p. Since this is a mean field approach, plus we have made some simplifications for the potential, we should expect the model to work in a reasonable way for average properties such as collective

flow and particles production after a suitable collision term is added to the Vlasov equation. Of course we cannot demand that light nucleon clusters and even nucleons will be formed with the correct quarks content. This is in many ways analogue to the situation in nuclear physics where one describes HIC around the Fermi energy using the Boltzmann Nordheim Vlasov equation [4]: average properties are reasonably reproduced such as collective flow, pion, photon, kaon..production, but to reproduce deuteron, triton, and other light heavy ions yields is clearly outside the reach of the model;

ii) in principle one should be able to obtain self-consistently DS in the Vlasov approach. However, if we give a different color charge to the t.p. in such a way that the total color charge is zero and make the bare Richardson's potential become repulsive for two equal charges then, due to the large number of t.p. which results in a smooth quark density in the box, the resulting average potential will be zero. This could be corrected by using a different way to initialize the system (which we have been unable to do so far) or use a molecular dynamics approach. There could be however some problems in molecular dynamics as well, such as numerical (the CPU time increases quadratically with the number of particles- at variance with the VE which is linearly dependent). Most important it will be very difficult to keep antisymmetrization at all times since molecular dynamics will evolve the system to its classical equilibrium state;

iii) one could also wonder if in our matter calculations, drops describing a confined nucleon, say at g.s. density, are really formed. This is not easy to see in the calculations since there is a time evolution and blobs of matter are created and destroyed at all times and only *on average* (over time) the density is constant. However, since the EOS we obtain is the same i.e. same density, binding energy and (by opportunely tuning the parameters) same compressibility of NM, such a problem does not apply.

## V. CONCLUSIONS

In conclusion in this work we have calculated the EOS of quark matter within a Vlasov approach at finite baryon densities and zero temperatures. We have shown that a suitable choice of the parameters and quark masses reproduces the nuclear matter properties near the ground state. One set of parameters (I) used is also able to reproduce hadrons masses as well. At large densities the EOS becomes softer than as given by a Skyrme force because of the transition to the QGP. The transition can be estimated comparing our results to the bag pressure and energy density. The density of the transition is close to the point where the  $c\bar{c}$  mesons dissolve in QM. However, this cannot be considered as a probe of the transition because other mesons such as  $u\bar{d}$  and heavy  $b\bar{b}$  dissolve at much higher density. Thus, at least within this approach, there is no first order phase transition and the transition from NM to QGP is smooth. The meson masses change depending on density and they disappear at high enough densities due to the repulsive parts of the potential and the Fermi motion which overcome the attractive Coulombic plus chromomagnetic potentials. The  $\rho$  meson, for which the c.t. is repulsive, dissolves at all densities in QM. Experiments in p(or e, $\gamma$ ,  $\pi$ ) -Nucleus collisions, where we expect the density to be close at  $\rho_0$  and T=0, can give some further constraint on the physics involved.



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- [14] Such large values of  $\alpha_0$  should not surprise. Infact recall that a QCD result [2] gives  $\alpha(Q^2) \propto 1/\ln(Q^2/\Lambda^2)$ . In QM the momentum transfer  $Q \approx p_f \approx \Lambda$  at  $\rho_0$ , thus we are in the region where the strong coupling constant can be very large.
- [15] For the sets of parameters I and II we can easily fit the masses of the  $\pi$  and the  $n$  in vacuum by tuning opportunely the values of  $\bar{\alpha}$  and  $\sigma$  in eq.(3). However, set I is able to reproduce the masses of the  $\rho$  and the  $\Delta$  as well, see sect.III [3], while set II gives masses 500-700 MeV larger than data for those particles. Heavier meson masses in vacuum are reproduced by fitting the corresponding quark masses since the c.t. gives a vanishing contribution, eq.(3) [3].
- [16] For set II  $\beta = 3.8$ , the  $(m_c, \rho_c)$  values are (9.79 GeV,  $17.15 \text{ fm}^{-3}$ ), (3.172 GeV,  $0.798 \text{ fm}^{-3}$ ), (3.174 GeV,  $0.8 \text{ fm}^{-3}$ ), (0.199 GeV,  $2.14 \text{ fm}^{-3}$ ), for the  $\gamma(1S)$ ,  $\eta_c(1S)$ ,  $J/\psi$  and the  $\pi$  respectively. For set I the results are qualitatively the same.

**Table I**

Quark masses used in this work.

<i>QuarkMass</i>	GeV
<i>u</i>	0.13
<i>d</i>	0.13
<i>s</i>	0.35
<i>c</i>	1.45
<i>b</i>	4.8
<i>t</i>	180.

# FIGURES

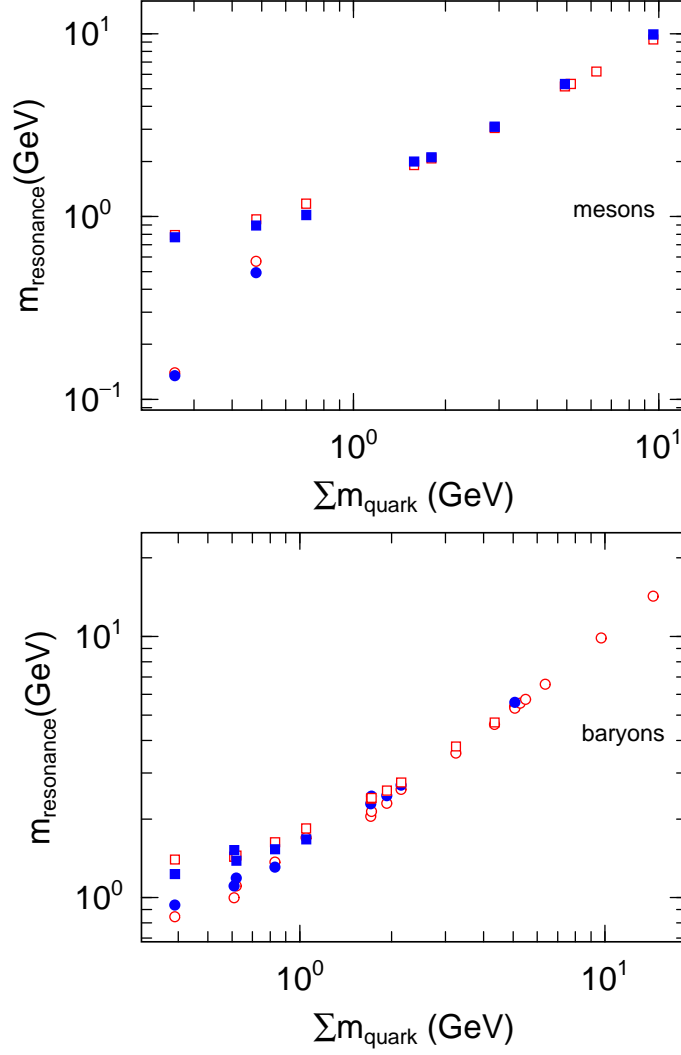


FIG. 1. Meson masses (top) vs. masses of the  $q\bar{q}$  pair. The symbols refer to the observed (full symbols) and calculated (open symbols) masses. The circles refer to the pseudoscalar and the squares to vector mesons masses. Similarly for baryons (bottom). Data are taken from [12].

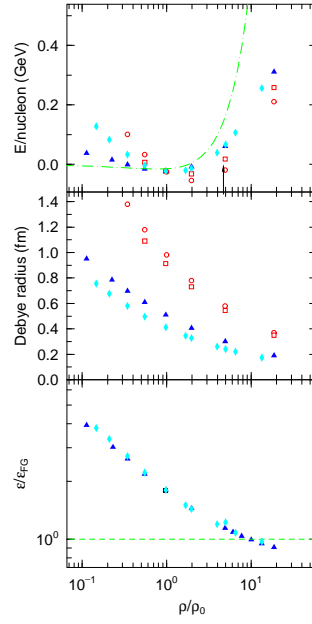


FIG. 2. Energy per nucleon (top), Debye radius (middle) and energy density of the quarks (bottom) vs. normalized density for various choices of the input parameters (see text). The dashed-dotted curve (top) is obtained using a Skyrme potential.

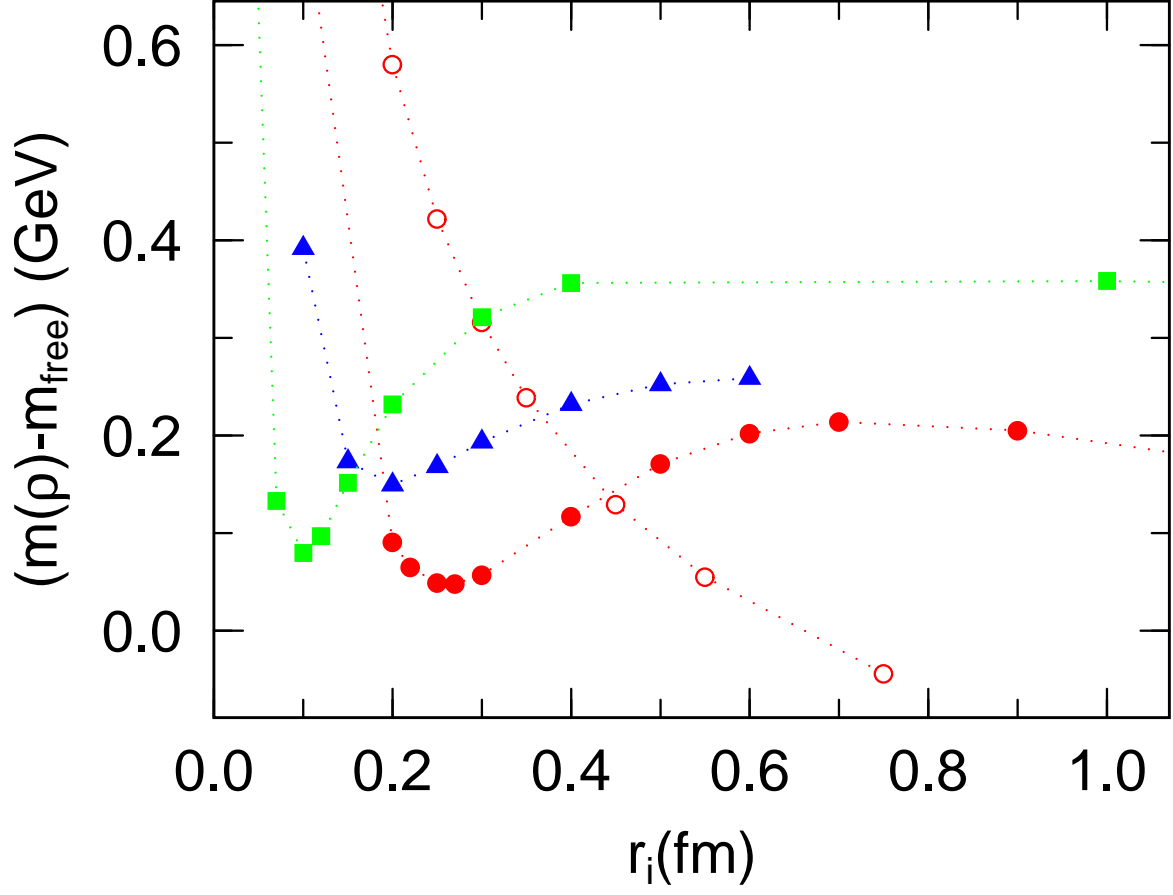


FIG. 3. Meson masses in medium (minus their free masses) vs the initial radius of the  $q\bar{q}$  systems. The open circles refer to the  $\rho$  meson at  $\rho = 0.0235 fm^{-3}$ , full circles and triangles refer to the  $\pi$  and  $\eta$  at  $\rho_0$ , the squares to  $\gamma(1s)$  at  $\rho = 1.046 fm^{-3}$ .

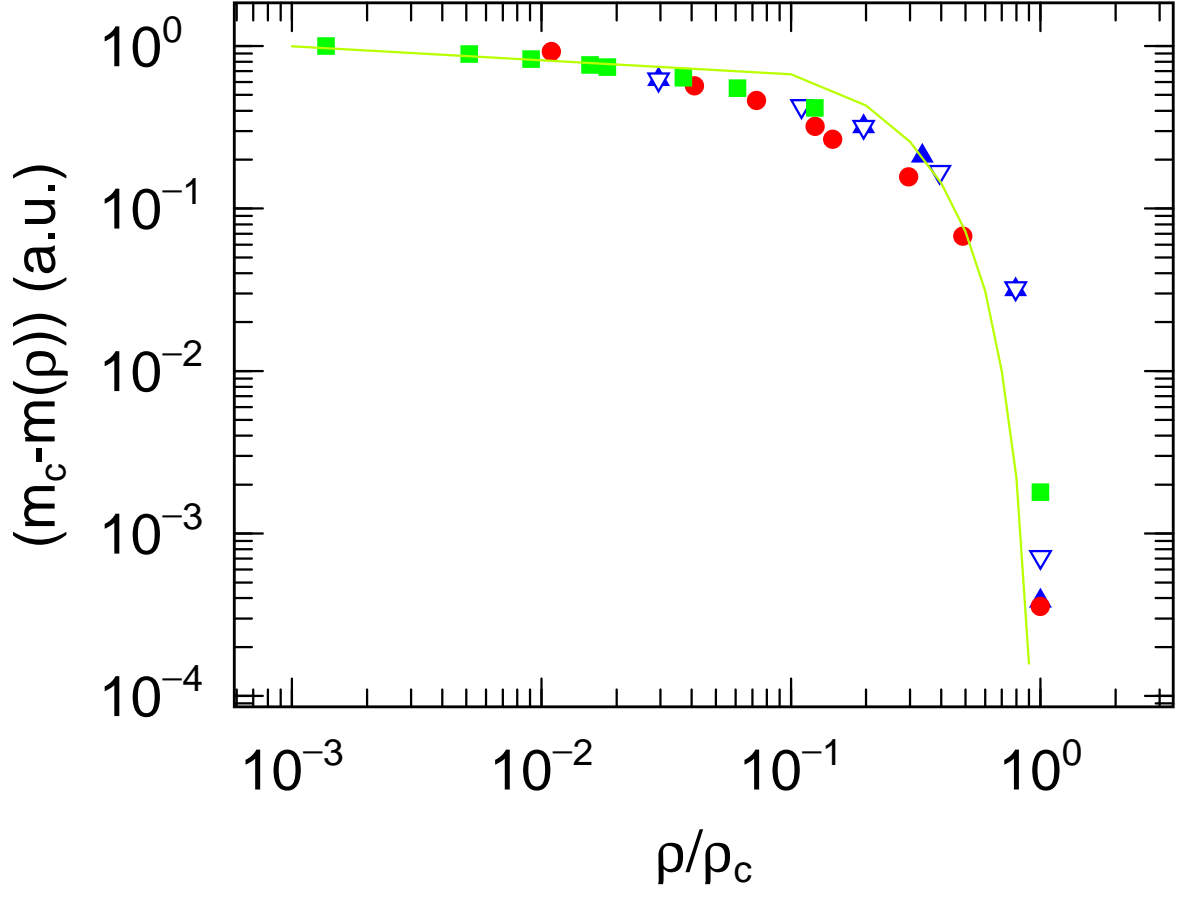


FIG. 4. 'Critical' minus meson mass calculated at  $\rho$  vs.  $\rho/\rho_c$  for  $b\bar{b}$ (squares),  $c\bar{c}$  (triangles) and  $u\bar{d}$  (circles). The curve is a fit according to eq.(1).